Neural Network-Based Numerical Methods for One-Dimensional Conservation Laws

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Abstract

The numerical approximation of nonlinear conservation laws relies on structure preserving finite volume schemes whose accuracy and robustness depend on the choice of reconstruction procedures and other heuristic design components. In recent years, physics-informed neural networks [1] have provided fully data-driven solvers, while several studies incorporated machine learning into high-resolution schemes, for instance by improving WENO reconstructions [2, 3] or constructing neural network-based limiters [4].

In this work, we propose a neural network architecture that parametrizes a family of TVD slope limiters within a conservative finite volume framework for one dimensional hyperbolic conservation laws. Rather than replacing classical schemes, the network mimics a structured class of methods and selects the optimal member during the training process. The architecture is derived directly from the limiter formulation, yielding an explicit one-to-one correspondence between the trainable network parameters and the coefficients of the induced numerical method. In this sense, the approach provides a minimal and interpretable parametrization of the admissible method class while preserving consistency and conservation. Numerical experiments are presented for the Burgers equation, a traffic flow model, and the shallow water system in one space dimension. The results demonstrate improved accuracy compared to standard limiter choices while maintaining stability properties of the underlying finite volume scheme.

References

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