Low-Rank Surrogates for Parametric PDEs

Benno Huber, Robert Scheichl, Jakob Zech

Heidelberg University, Heidelberg, Germany
BENNO.HUBER@UNI-HEIDELBERG.DE

Abstract

Evaluating the forward solution operator for parametric partial differential equations (PDEs) typically involves solving large linear systems, making the process computationally expensive. Operator surrogates aim to provide a computationally efficient approximation to these parameter-to-solution operators. This is particularly beneficial in applications requiring many evaluations of the forward map, such as uncertainty quantification (UQ) or optimization.

Mathematically, operator surrogates provide approximations of mappings between infinite dimensional spaces. We analyze an encoder-decoder framework, where the input and output function spaces are parametrized using coefficient sequences of admissable representation systems. The goal then becomes to approximate a coefficient-to-coefficient map, which can be realized using various approximation tools. In the present work we focus on low-rank tensor representation using the tensor-train (TT) format. This format allows for efficient storage, evaluation and basic linear algebra as long as the tensor ranks remain low. Its structure also allows for efficient quadrature, useful e.g. in UQ applications. We show approximation rates with regards to the storage complexity of such TT surrogates for certain holomorphic maps by providing rank bounds. Additionally, we present benchmark results comparing the TT surrogate to a range of other operator surrogate methods applied to a parametric diffusion problem with varying input smoothness.