

Discovering Adaptive Basis Representations for Parametrized PDEs with Deep Orthogonal Decomposition: Theory and Practice

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Abstract

We present Deep Orthogonal Decomposition (DOD) [1], a novel technique for dimensionality reduction and reduced order modeling of parametrized PDEs. The DOD consists in a deep neural network approximating the solution manifold through a continuously adaptive local basis. In contrast to global techniques such as Proper Orthogonal Decomposition (POD), the local adaptivity of the learned basis allows DOD to mitigate the Kolmogorov barrier, significantly broadening its applicability to challenging nonlinear problems. Additionally, thanks to the orthogonal structure of the latent space, DOD ensures a tight control on error propagation and enhanced interpretability, resulting in an appealing alternative to deep autoencoders. Beyond steady parametric settings, the proposed framework naturally extends to time-dependent PDEs by treating time as an additional input variable, allowing the learned basis to adapt continuously to the evolving system state. From this perspective, DOD can be interpreted as a data-driven tool for learning low-dimensional representations of nonlinear dynamical systems. The methodology is analyzed both theoretically and practically. On the one hand, we establish a connection between the truncation error of the DOD and a spectral gap condition related to the solution manifold [2], whereas on the other we assess the performances of the DOD on a set of numerical experiments involving nonlinear PDEs with parametrized geometries and high-dimensional parameter spaces [1].

References

- [1] N. R. Franco, A. Manzoni, P. Zunino, and J. S. Hesthaven, Deep orthogonal decomposition: a continuously adaptive neural network approach to model order reduction of parametrized partial differential equations. *Advances in Computational Mathematics*, accepted, 2026.
- [2] N. R. Franco, Measurability and continuity of parametric low-rank approximation in Hilbert spaces: linear operators and random variables. *Revista Matemática Complutense*, pages 1–42, 2025.