

Learning the Continuous-Time Dynamics: from Trajectories to Velocities

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Abstract

Learning nonlinear continuous-time dynamical systems is a central problem in many fields of science and engineering. Deep learning architectures characterized by a continuous-time inductive bias, such as Neural ODEs, have seen widespread adoption in this context, with applications ranging from low-dimensional dynamical systems modeling to data-driven order reduction for time-dependent PDEs by relying on suitable nonlinear dimensionality reduction strategies. Despite many advantages stemming from their continuous-time inductive bias, including mathematical interpretability and time super-resolution, they rely on a simulation-based training procedure, which, whether employed directly in state-space or in a latent space of reduced dimension, requires unrolling the predictions over multiple steps by means of numerical integration. Rollout-based training, while motivated by empirical evidence for providing stable predictions, involves high computational costs and memory requirements due to backpropagation through time, which are further compounded by higher-order numerical integration of the Neural ODE. In this talk, we first address the pitfalls of rollout-based training in the context of learning continuous-time dynamics, analyzing the bias introduced by the numerical solver when unrolling predictions in the infinite-horizon limit, thereby hindering proper identification of the underlying dynamics. Then, we discuss the advantages of a velocity-based training objective, by proposing the adoption of a stochastic objective that results in a higher-order approximation of the population risk, and is characterized by an implicit regularization effect. Numerical experiments, carried out in the context of dynamical systems and time-dependent PDEs, validate the efficiency of the proposed approach, highlighting faster convergence and improved generalization compared to a range of rollout-based training strategies.

References

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