Deep Symmetric Autoencoders: Error Estimates and Beyond

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Abstract

Within this talk, we aim to bridge deep autoencoders and proper orthogonal decomposition (POD) through deep symmetric autoencoders, which take the best of both worlds. Indeed, deep symmetric autoencoders retain the nonlinear nature of classical deep autoencoders, which is central to overcome the limitations of linear reduction methods when tackling problems with a slow decay of the Kolmogorov n-width. Furthermore, akin to linear reduction methods, deep symmetric autoencoders are endowed with a rich mathematical structure, which is the perfect ground to devise clear and natural error estimates [1], by exploiting the powerful Eckart-Young-Schmidt theorem [2] and the properties of the employed activation function [3]. Nonetheless, leveraging our theoretical framework and the proposed error estimates, we derive a data-driven initialization procedure for deep symmetric autoencoders, which entails the iterative application of POD at each layer. Finally, we discuss the practical aspects of our work through a series of numerical experiments, aimed to show the representation capacity of deep symmetric autoencoders and the effectiveness of our proposed initialization strategy.

References

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