Acceleration of a Nonoverlapping Domain Decomposition Solver by Using GPUs on Subdomains

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Abstract

Balancing Domain Decomposition based on Constraints (BDDC) is a method for solving large sparse linear systems of equations arising from the discretization of partial differential equations (PDEs). It is based on using the conjugate gradient (CG) method preconditioned by a two-level preconditioner based on nonoverlapping domain decomposition. Since its introduction in 2003, BDDC has been successfully applied to a wide range of problems, including structural mechanics, fluid dynamics, and electromagnetics. We have implemented the multilevel extension of BDDC using adaptive selection of the coarse problem in the open-source library BDDCML[1].

In domain decomposition methods, the dominant cost is associated to the solution of local problems on subdomains, which are independent and can be solved in parallel. In BDDC, there are two local problems to be solved within each CG iteration: i) a local problem associated to the multiplication of the search direction vector by the Schur complement at the interface, and ii) a local subdomain correction problem within the preconditioner. In [1], an approach to accelerating these two dominant components of the solver by using graphics processing units (GPUs) was investigated. It is based on building dense matrices of the local Schur complements by the sparse direct solver on CPUs in the preconditioner setup. In the next step, the local matrices are copied to GPUs. Repeated multiplications of local vectors with the dense matrix of the Schur complement are performed for each subdomain on GPUs. In addition, factorizations and backsubstitutions with the dense saddle-point subdomain matrices of the subdomain correction problems are also performed on GPUs.

In this contribution, detailed times of the main components of the algorithm are measured on a benchmark Poisson problem on the LUMI supercomputer. The method is also applied to an unsteady problem of incompressible flow, where the Krylov subspace iterations are performed repeatedly in each time step. The results demonstrate the potential of the approach to speed up realistic simulations more than 10 times with a preference towards large subdomains.

References

[1] J. Šístek, T. Oberhuber, Acceleration of a parallel BDDC solver by using graphics processing units on subdomains, The International Journal of High Performance Computing Applications 37 (2) (2023) 151–164.

[1] `https://users.math.cas.cz/~sistek/software/bddcml.html`