Low-Rank Methods in Isogeometric Analysis

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Abstract

Isogeometric Analysis (IgA) bridges CAD and finite element analysis by employing spline technologies (typically B-splines or NURBS) for both geometry representation and field approximation, enabling exact geometrical descriptions and high-order smooth trial spaces. These advantages come with computational challenges: higher-order spline bases have supports spanning multiple knot spans, yielding dense element contributions in global system matrices and expensive multi-dimensional quadrature in tensor-product settings [1]. To address these bottlenecks, low-rank tensor techniques exploit the intrinsic tensor-product structure of IgA. The core idea is to separate variables in geometry-induced integral kernels and represent mass and stiffness tensors as sums of Kronecker products of univariate matrices, reducing assembly cost and memory footprint while maintaining accuracy control. Building on this, Bünger et al. introduced tensor-train (TT) approximations for coefficient tensors arising from kernel interpolation in tensor-product B-spline bases, computed via the Alternating Minimal Energy (AMEn) method [2]. The talk then presents two extensions of this TT-based paradigm from our recent work. First, we show how multi-patch IgA can be treated by transferring the IETI (isogeometric tearing and interconnecting) concept to the low-rank TT framework and enforcing interface continuity via dual Lagrange-multiplier constraints [3]. Second, we summarize a hierarchical refinement strategy based on (T)HB-splines, where separability is recovered on structured subregions to enable level-wise low-rank assembly and block-TT solution procedures on locally refined meshes. A remaining issue is that interpolating geometry-induced kernels in tensor-product spline spaces produces global coefficient tensors: such global parameterizations struggle to capture local features and cannot naturally represent discontinuities or singularities in complex geometries. As an outlook, we discuss how functional tensor trains may provide a route to local refinement and non-smooth phenomena without relying on a fixed tensorized discretization.

References

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