Hybrid Hierarchical Matrices with Adaptive Precision Storage

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Abstract

Hierarchical matrices are multilevel block low-rank representations of matrices that are widely used for fast matrix computations. Hierarchical matrices are built using a tree data structure with low-rank blocks identified using various admissibility conditions, such as standard/strong admissibility and weak admissibility. In this talk, we introduce a novel hierarchical matrix framework, namely $\mathcal{H}_h$, based on a *hybrid* admissibility condition: we use strong admissibility at the coarser levels (larger blocks) and weak admissibility at the finer levels (smaller blocks). We carry out a rounding error analysis of $\mathcal{H}_h$-matrices and show that the low-rank blocks of $\mathcal{H}_h$-matrices can be represented in low precision without degrading the overall approximation quality. We provide an explicit rule for choosing the precision for a particular low-rank block. We propose an adaptive mixed precision algorithm for constructing and storing $\mathcal{H}_h$-matrices. Our numerical results illustrate that the proposed adaptive mixed precision $\mathcal{H}_h$-matrices can achieve significant storage reductions (up to $11\times$) compared to uniform double precision $\mathcal{H}$-matrices, without compromising accuracy.