

Dirichlet-Neumann Averaging: Efficient SPDE-Based Gaussian Process Simulation

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Abstract

Gaussian processes (GPs) and Gaussian random fields (GRFs) are essential for modelling spatially varying stochastic phenomena. Yet, the efficient generation of corresponding realisations on high-resolution grids remains challenging, particularly when a large number of realisations are required. In this talk I will present two novel contributions. First, we propose a new methodology based on Dirichlet-Neumann averaging (DNA) to generate GPs and GRFs with isotropic covariance on regularly spaced grids. The combination of discrete cosine and sine transforms in the DNA sampling approach allows for rapid evaluations without the need for modification or padding of the desired covariance function. While this introduces an error in the covariance, our numerical experiments show that this error is negligible for most relevant applications, representing a trade-off between efficiency and precision. We provide explicit error estimates for Matern covariances. The second contribution links our new methodology to the stochastic partial differential equation (SPDE) approach for sampling GRFs. We demonstrate that the concepts developed in our methodology can also guide the selection of boundary conditions in the SPDE framework. We prove that averaging specific GRFs sampled via the SPDE approach yields genuinely isotropic realisations without domain extension, with the error bounds established in the first part remaining valid. We finish by presenting an application within a parallel high-performance budgeted multi-level Monte Carlo method for estimates on the entire spatial domain of multi-PDE problems with random input data, demonstrating the advantages of the DNA approach in terms of ease of implementation, memory efficiency and parallel scalability. This is joint work with Robert Kutri and Niklas Baumgarten (Heidelberg).