

A Reynolds-Semi-Robust $H(\text{div})$ -Conforming Method for Unsteady Incompressible Power-Law Flows

L. Beirão da Veiga^{*}, D.A. Di Pietro[†] and K.B. Haile^{*}

^{*} Dipartimento di Matematica e Applicazioni Università di
Milano Bicocca, Milano, Italy
LOURENCO.BEIRAO@UNIMIB.IT, K.HAILE@CAMPUS.UNIMIB.IT

[†] IMAG, Univ. Montpellier, CNRS, Montpellier, France
DANIELE.DI-PIETRO@UMONTPELLIER.FR

Abstract

In this work, we address the numerical approximation of unsteady systems of p-Navier–Stokes type equations that model incompressible fluid flows with non-Newtonian rheology. The interplay between the nonlinearity of the non-coercive convective term, typical of Navier–Stokes, and the nonlinearity introduced by the non-Newtonian constitutive law presents interesting challenges. In the present talk, based on [3], we introduce a new $H(\text{div})$ -conforming, ”convectionsemi-robust” and pressure-robust method for solving unsteady systems of p-Navier–Stokes type equations. A numerical scheme is considered ”convection-semi-robust” if its velocity error estimates are independent of the inverse of the kinematic viscosity, while pressure robustness ensures that the velocity error estimates are unaffected by the pressure. These two conditions are considered critically important in modern literature (see e.g., [1]). To derive sharp local estimates, the complex interaction between the nonlinear convective term and the nonlinear diffusion term must be carefully analyzed. This interaction cannot be addressed through a straightforward combination of existing techniques, as the estimation of each term depends on the local flow regime. The peculiarity of our error estimates is that they track the dependence of the local contributions to the error on local Reynolds numbers, in the spirit of [2] developed for the scalar problem, which combines p-laplacian diffusion with convection and reaction terms. In the present talk, after presenting the model and the numerical method, we will outline the theoretical results. Finally, a set of numerical tests supporting the theory will be shown

References

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- [2] L. Beirão da Veiga, D. A. Di Pietro, and K. B. Haile, A Péclet-robust discontinuous Galerkin method for nonlinear diffusion with advection, *Math. Models Methods Appl. Sci.* (2024) 34.9: 1781–1807.
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