A High-Order Discontinuous Galerkin Method for the Numerical Modeling of Neuronal Electro-Metabolic Dynamics

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Abstract

Numerical approximation of traveling wave-like phenomena represents a challenging problem in the simulation of multiscale processes. These kinds of models characterize various biological scenarios, such as electrical propagation in excitable tissues, and they require numerical methods that combine high spatial and temporal accuracy with geometric flexibility, due to the presence of heterogeneous tissues and sharp wavefronts. High-order discontinuous Galerkin methods on polytopal and polyhedral meshes (polyDG) [1, 4] provide a particularly suitable framework for the discretization of these problems. In particular, the use of high-order polynomials enables an accurate resolution of steep and fast-propagating wavefronts, while the element-wise structure of DG methods naturally supports p-adaptive strategies [3], allowing the local polynomial degree to be adjusted dynamically where the solution exhibits steep gradients, in order to improve computational efficiency. Furthermore, this formulation offers significant geometric flexibility for the treatment of complex computational domains. A relevant application of these models arises in the electrophysiological modeling of brain tissue. At the tissue scale, neuronal activity can be described by the monodomain equation, coupled with appropriate ionic and metabolic models that account for underlying metabolic dynamics and ion-channel mechanisms [2]. Within this framework, we investigate how oxygen availability influences cellular energy production and ionic pump activity under pathological hypoxic conditions, providing a mathematical description of the interaction between electrical activity and metabolic processes.

References

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