

An Introduction to Computer-Assisted Proofs in Differential Equations

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Abstract

Computer-assisted proofs (CAPs) have undergone remarkable development in recent years, fueled by advances in functional-analytic frameworks for infinite-dimensional dynamical systems, reliable interval-arithmetic numerical methods, and high-performance computing. These techniques now make it possible to rigorously validate equilibria, periodic orbits, invariant manifolds, connecting orbits, and even chaotic dynamics in ordinary, delay, and partial differential equations, often in regimes far beyond the reach of classical analytical methods. In this introductory talk, I will present the general philosophy and methodology behind CAPs, focusing on a Newton–Kantorovich framework for the validation of solutions. The emphasis will be on the core ideas: how numerical approximations can be combined with functional analysis and rigorous error bounds to produce mathematically certified results. The goal is to provide a clear conceptual roadmap of the approach, illustrated with representative examples.