Computer-Assisted Proofs for PDEs: a Matrix-Free Approach

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Abstract

Computer-assisted proofs have become more and more popular in the study of differential equations and other dynamical systems [1]. Many of them revolve around one central idea: first obtain numerically an approximate solution of your problem, and then prove, via a fixed point argument, the existence of a true solution nearby. To that end, it is often crucial to build an accurate approximation of the inverse of the linearized operator, which is obtained by combining quantitative compactness estimates and the numerical computation of a finite dimensional approximate inverse. Once quantitative compactness estimates are available, one should *in principle* be able to perform a computer-assisted proof. However, *in practice*, one must still be able to deal with the finite dimensional approximate inverse. For 2D or 3D PDEs, this is often where the bottleneck of such proofs lies.

In this talk, I will describe in more details the general strategy outlined above, and then present a new matrix-free approach to deal with the finite dimensional part, which uses only standard ideas from scientific computing and numerical linear algebra, but scales better than the usual approach used in most computer-assisted proofs. This is joint work with Hugo Chu (Ecole polytechnique).

References

[1] J. B. van den Berg and J.-P. Lessard. Rigorous numerics in dynamics. *Notices Amer. Math. Soc.*, 62(9), 2015.