On the Singularities in Area-Preserving Curvature Flows of Convex Symmetric Immersed Closed Plane Curves

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Abstract

In this talk, we consider the motion of the convex immersed plane curves, which is described by the following differential equation with a nonlocal term $\Lambda_p(t)$;

$$v_t(\theta,t) = v(\theta,t)^p\big(v_{\theta\theta}(\theta,t) + v(\theta,t) - \Lambda_p(t)\big), \quad \theta \in (-m\pi, m\pi),$$

$$\Lambda_p(t) := \left(\int_{-m\pi}^{m\pi} \frac{1}{v(\theta,t)^{p-1}}\,d\theta\right)^{-1} \int_{-m\pi}^{m\pi} \frac{1}{v(\theta,t)^{p-2}}\,d\theta.$$

Here, p is a parameter, and the relation between v and the curvature k of the curve is given by $v(\theta,t) = (p-1)^{-\frac{1}{p}} k(\theta,t)^{\frac{1}{p-1}}$. Due to the effect of the nonlocal term, the enclosed area of the curve is conserved. This motion of the curve is called area-preserving curvature flow. This problem was first considered in [2] and the development of the finite time singularity was conjectured ($p = 1$). For Abresch-Langer type curves with highly symmetric properties, it has been proved that the maximum of curvatures blows up at a finite time under some assumptions [4, 3] ($p \geq 1$). In this case, we consider the blow-up rate.

This talk is based on joint work with Koichi Anada and Tetsuya Ishiwata [1].

References

[1] Koichi Anada and Tetsuya Ishiwata and Takeo Ushijima, Type II singularities in area-preserving curvature flows of convex symmetric immersed closed plane curves, Journal of Differential Equations 437 (2025) 113348.

[2] M.E. Gage, On an area-preserving evolution equation for plane curves, Nonlinear Problems in Geometry, Contemp. Math., Amer. Math. Soc. 51 (1985) 51–62.

[3] N. Sesum, D.H. Tsai, X.L. Wang, Evolution of locally convex closed curves in the area-preserving and length-preserving curvature flows, Comm. Anal. Geom. 28 (2020) 1863–1894.

[4] X.L. Wang, L.H. Kong, Area-preserving evolution of nonsimple symmetric plane curves, J. Evol. Equ. 14 (2014) 387–401