Existence and Uniqueness of L^1-Solutions to Time-Fractional Nonlinear Diffusion Equations

Tatsuki Kawakami

Ryukoku University, Otsu, Japan
KAWAKAMI@MATH.RYUKOKU.AC.JP

Abstract

Consider the Cauchy problem for time-fractional porous medium type nonlinear diffusion equations

$$\begin{cases} \partial_t \left(g_\alpha * [u - u_0]\right) = \Delta |u|^{m-1} u, & x \in \mathbb{R}^N, \quad t > 0, \\ u(x, 0) = u_0(x), & x \in \mathbb{R}^N, \end{cases} \tag{P}$$

where $N \geq 1$, $\alpha \in (0, 1)$, $m > 0$, and u_0 is an integrable function in $\mathbb{R}^N$. Here

$$g_\alpha(t) := \frac{t^{-\alpha}}{\Gamma(1-\alpha)},$$

which is so-called the Riemann-Liouville kernel, and $\partial_t^\alpha (u - u_0)$ is so-called the α-th order Caputo derivative of u if it is sufficiently smooth. In this case, the equation (P) is so-called the fast diffusion equation if $0 < m < 1$, the heat equation if $m = 1$, and the porous medium equation if $m > 1$. In this talk, under the condition (K) and (L), we establish the global existence and uniqueness of L^1-solutions to problem (P). Furthermore, we give the mass conservation law for L^1-solutions to time-fractional fast diffusion equations, namely $0 < m < 1$, and prove that the finite-time extinction does not occur for any nonnegative L^1-solutions.

This talk is based on a joint work with Mikiya Kametaka (Ryukoku University).