

On an Optimal Strict Sturm Majorant Condition for Second Order Linear Differential Equations

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Abstract

This is a joint work with Roman Šimon Hilscher. Given a real interval $\mathcal{I}$ and continuous functions $\hat{r}, \hat{p}, r, p : \mathcal{I} \rightarrow \mathbb{R}$ we consider two second order linear differential equations

$$(\hat{r}(t)\hat{x}')' + \hat{p}(t)\hat{x} = 0, \quad (r(t)x')' + p(t)x = 0, \quad t \in \mathcal{I}. \quad (1)$$

together with the majorant condition

$$\hat{p}(t) \leq p(t), \quad \hat{r}(t) \geq r(t) > 0, \quad t \in \mathcal{I}. \quad (2)$$

In this talk we present a new type of condition (2) for equations in (1) on a general noncompact (possibly unbounded) interval $\mathcal{I}$ derived in [6]. This can be viewed as an-extension of the classical result by Coppel in [3] for the compact interval $\mathcal{I}$ as well as an-extension of the result of Aharonov and Elias in [1, 2] for the noncompact interval $\mathcal{I}$ under assumption $r(t) = \hat{r}(t)$ on $\mathcal{I}$. We will apply a singular Sturmian theory for possibly uncontrollable linear Hamiltonian systems, which was recently developed in [4, 5]

References

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