

Koopman Semigroups on Lebesgue Spaces

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Abstract

Composition operators in dynamic systems are often referred to as Koopman operators, in honor of the French-American mathematician Bernard Osgood Koopman (1900–1981). In this talk, we present a brief introduction to Koopman semigroups. We then introduce three examples of weighted Koopman semigroups defined on fractional Lebesgue-Sobolev spaces on the real ray.

We are also interested in connecting Koopman semigroups in Lebesgue spaces of functions $L^p(\mathbb{R}^+)$ and semigroups in Lebesgue spaces of sequences ℓ^p for $1 \leq p < \infty$. To achieve this, we use a certain Poisson transformation $\mathcal{P} : L^p(\mathbb{R}^+) \rightarrow \ell^p$ and its adjoint $\mathcal{P}^*$, which allows us to transfer the semigroup properties from one space to another. Two Koopman semigroups in ℓ^p are presented and related to the canonical Koopman semigroup in $L^p(\mathbb{R}^+)$.

In the last part of the talk, we introduce operators that extend Cesàro operators (called Chen-type integral operators) subordinate to these Koopman semigroups in $L^p(\mathbb{R}^+)$ and ℓ^p .

The first results of this article are joint work with Verónica Poblete from the University of Chile and are published in Monatshefte für Mathematik, 206, (2025). The second part of the talk contains results from a preprint available on the Arxiv platform.

References

- [1] P. J. Miana and V. Poblete, *Three weight Koopman semigroups on Lebesgue space*, Monatsh. für Math. (2025) <https://doi.org/10.1007/s00605-024-02041-2>
- [2] P. J. Miana, *Two Koopman semigroups on discrete Lebesgue spaces*. ArXiv:2511.13868 (2025).