Period Two Implies Chaos for a Class of ODEs Revisited

Jan Andres

Palacký University, Olomouc, Czech Republic
JAN.ANDRES@UPOL.CZ

Abstract

It is well known that neither the celebrated Sharkovsky cycle coexistence theorem, nor Li-Yorke's "Period three implies chaos" can be applied in their standard forms to differential equations. On the other hand, Obersnel and Omari (cf. [3]) formulated, in reaction to our earlier results, the Sharkovsky-type theorem for the scalar ODEs, but without Sharkovsky's ordering and no exceptional cases. Nevertheless, despite a rather provocative title of [3], there is no related information about chaos at all.

The main aim of our talk is therefore to deduce criteria of chaos in terms of the associated multivalued Poincaré translation operators along the trajectories of given scalar ordinary differential equations and inclusions. Besides other things, we will show that such criteria essentially depend on the definitions of various sorts of chaos. For instance, topological entropy can be at the same time infinite in the sense of one definition and zero in the sense of another.

As far as we know, some particular results along these lines were obtained just by Pireddu (cf. [4]) in terms of the Bebutov flows, in [1] via single-valued chaotic selections and more recently in [2] for hidden orbits of discontinuous maps.

References

[1] Andres, Jan. Period two implies chaos for a class of multivalued maps: a naive approach. *Comput. Math. Appl.* **64** (2012), no. 7, 2160–2165.

[2] Jeffrey, Mike R.; Avrutin, Viktor. Periods $1 + 2$ imply chaos in steep or nonsmooth maps. *Nonlinearity* **35** (2022), no. 12, 6014–6041.

[3] Obersnel, Franco; Omari, Pierpaolo. Period two implies chaos for a class of ODEs. *Proc. Amer. Math. Soc.* **135** (2007), no. 7, 2055–2058.

[4] Pireddu, Marina. Period two implies chaos for a class of ODEs: a dynamical system approach. *Rend. Istit. Mat. Univ. Trieste* **41** (2009), 43–54 (2010).