Dynamics of a Piecewise Smooth Ghil-Zaliapin-Thompson ENSO Model

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Abstract

The Ghil–Zaliapin–Thompson (GZT) model, a scalar delay differential equation with periodic forcing and time-delayed feedback, captures key features of the El Niño–Southern Oscillation (ENSO) phenomenon. Numerical studies have revealed parameter regimes of very complex dynamics, but its analytical treatment remains challenging. To bridge this gap, we propose [1,2] a piecewise smooth version of the GZT model

$$h'(t) = -\text{sign}(h(t-\tau)) + c\cos(2\pi t),$$

with piecewise constant delayed feedback and continuous periodic forcing.

For this piecewise smooth GZT model we explicitly construct n:s-periodic orbits of odd integer period n with s upward crossings of zero, and derive existence and stability conditions for them, and identify the bifurcations that delimit their regions of existence in the (τ, c)-parameter plane.

For $c \gg 0$ we show that there is a unique stable 1:1-periodic orbit (seasonal forcing dominates), which loses stability along a bifurcation curve that we identify explicitly. For most values of τ this occurs in a torus bifurcation. For small c the dynamics is altogether more interesting with overlapping resonances tongues composed of n:s and n:$(s+2)$-periodic orbits with n and s odd and $s \leq (n-1)/2$. All of the period n orbits that we study satisfy $h(t) = -h(t+n/2)$, and this symmetry is central to our constructions and analysis.

This is joint work with Sam Bolduc-St-Aubin of the University of Auckland, New Zealand.

References

[1] S. Bolduc-St-Aubin and A.R. Humphries Seasonal Forcing Dominated Dynamics of a piecewise smooth Ghil-Zaliapin-Thompson ENSO model. arXiv:2510.27084 [math.DS] doi: 10.48550/arXiv.2510.27084 (2025)

[2] S. Bolduc-St-Aubin and A.R. Humphries Resonant Dynamics of a piecewise smooth Ghil-Zaliapin-Thompson ENSO model. In preparation (2026)