Co-Orbital Solutions in the Planar Three-Body Problem

Patricia Yanguas

Departamento de Estadística, Informática y Matemáticas,
Universidad Pública de Navarra, Pamplona, Spain
YANGUAS@UNAVARRA.ES

Abstract

The purpose of this work is to provide a qualitative explanation of the co-orbital motion of two small moons orbiting a common planet.

The two moons revolve around the planet along nearly circular trajectories with almost equal radii. The system is modelled as a planar three-body problem, whose Hamiltonian function is developed as a perturbation of two decoupled Kepler Hamiltonians. Techniques from averaging theory are combined with normal forms, symplectic scalings, and Hamiltonian reduction, together with the application of a suitable KAM theorem for highly degenerate cases.

The goal is to establish the persistence of 4-dimensional KAM tori and quasi-periodic solutions associated with the co-orbital motion of the moons. A region of the reduced phase space (which is the Cartesian product of a two-sphere and a two-sheeted hyperboloid of revolution) must be selected, and within this portion of phase space the Hamiltonian is expressed in suitable action-angle coordinates at successive orders of perturbation, up to fourth order, in order to apply KAM theory. One of the action variables is related to the angular momentum and can take any value, thereby extending previous work by the same authors.

Finally, applications to the motion of Janus and Epimetheus around Saturn are discussed, as well as the generalisation to the spatial three-body problem.

This work is a collaboration with Josép María Cors and Jesús F. Palacián.

References

[1] On co-orbital quasi-periodic motion in the three-body problem. J.M. Cors, J.F. Palacián, P. Yanguas, *SIAM Journal on Applied Dynamical Systems* (2019) 18, 334–353.