

Nekhoroshev Estimates for Multi-Scale Hamiltonian Systems

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Abstract

Motivated by applications arising in celestial mechanics, we give some Nekhoroshev stability estimates for multi-scale nearly-integrable Hamiltonian systems. Particular attention is paid to two special cases: (I) The integrable part at each scale is (α, K) completely non-resonant; and (II) the resonance only occurs in the highest scale of the integrable part. By considering non-resonance and obtaining normal forms via a finite number of steps of an averaging process, not only do we obtain the Nekhoroshev estimates for both cases, but also we reveal some impacts of multi-scales on the nature of stability. In particular, the estimates of the stability time are determined by the smallness of the perturbation in the normal form, while the stability radius is also influenced by the resonant gap in the second case. Moreover, in the non-resonant case, the estimates of each component are relatively independent, while in the resonant case, the stability time is determined by both non-resonant and resonant terms. Some examples of multi-scale Hamiltonian systems arising as perturbations of the Kepler problem are given to illustrate the application of our results.