

Ejection-Collision Solutions and KAM Tori in Restricted N -Body Problems

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Abstract

We study ejection-collision solutions in several cases of the N -body problem, starting with the restricted three-body problem in the planar case. In these solutions the particle of negligible mass passes several times near one of the primaries before colliding with it. In the so-called lunar regime, the Hamiltonian is a perturbation of the Kepler problem. In a first step, applying the Han-Li-Yi theorem, we prove the persistence of KAM tori filled in with quasi-periodic motions of the small particle. This is achieved by using some techniques of Hamiltonian dynamics including regularisation of collision orbits, normalisations and symplectic reductions. Second, from the rectilinear quasi-periodic motions we identify those corresponding to the ejection-collision solutions. The analysis of these solutions in the planar case of the restricted three-body problem has been performed since the 1980s and more recently by M. Ollé and collaborators from both an analytical and a numerical point of view. Our approach is generalised to the planar restricted N -body problem and also to the spatial case of the restricted three-body problem. In the restricted N -body problem normally four families are found, but under some extra symmetries and degeneracies related to the configurations of the primaries, more families can be obtained. In the spatial restricted three-body problem, in addition to the four families found in the planar case, two new families of ejection-collision orbits are identified, that correspond to vertical motions of the negligible particle. The analysis of the spatial restricted three-body problem has been performed mainly numerically, hence the interest in applying the techniques mentioned for the search for analytical results.

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References

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