

Evolution problems with nonlocal equations and constraints: a survey

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Abstract

Nonlocal evolution problems arise naturally in the study of dynamical systems where the present state depends not only on instantaneous data but also on past or globally distributed information. Such nonlocal features may appear both in the evolution equation, through delay or history-dependent terms, and in the associated constraints, including multipoint, integral, or functional conditions. This survey presents an overview of abstract evolution problems involving various types of nonlocal equations and nonlocal constraints, with particular emphasis on the notion of the support of a nonlocal condition and its influence on solvability. Existence results are obtained by means of general fixed point techniques combined with the use of suitable equivalent Bielecki-type norms. For systems of equations, vectorial methods based on matrix estimates are employed, allowing componentwise information on the behavior of solutions. The presentation refers to some of the results obtained jointly with several collaborators, and suggests some perspectives for further developments.

References

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