On Impulsive Problems Driven by Second Order Differential Inclusions

Giulia Duricchi

University of Modena and Reggio Emilia
Department of Sciences and Methods for Engineering,
Reggio Emilia, Italy
G.DURICCHI@UNIMORE.IT

Abstract

The talk will be about new results, obtained in [1], [2] in collaboration with Tiziana Cardinali (Univ. of Perugia) and Valentina Taddei (Univ. of Modena and Reggio Emilia) on the existence of impulsive mild solutions for a problem driven by the following non-autonomous semilinear second order differential inclusion

$$x''(t) \in A(t)x(t) + F(t, x(t), x'(t)), \tag{1}$$

where $\{A(t)\}_t$ is a suitable family of operators generating a fundamental system (see [3]).
To establish this goal, we first investigate the existence of mild solutions for a Cauchy problem governed by the same differential inclusion (1) with initial value belonging to a suitable space E_γ, $\gamma \geq 0$. We introduce this space, by extending a classical set presented by Travis and Webb in order to study autonomous problems. In this setting we obtain continuously differentiable mild solutions even in absence of compactness or strong regularity assumptions on the multivalued nonlinearity.
Then, the impulsive problem is treated by constructing a subspace $E \subset E_\gamma$, and by requiring an evolution property on the fundamental system generated by the family $\{A(t)\}_t$. This assumption plays a key role in ensuring the regularity of the solutions and in using the gluing method to achieve the existence of impulsive mild solutions.
As an application of our results we study an impulsive problem, subjected to a control action given by a multimap U, monitored by the following hyperbolic integro-differential equation

$$\begin{aligned}\frac{\partial^2 w}{\partial t^2} =& \frac{\partial^2 w}{\partial \xi^2} + b(t,\xi)\frac{\partial w}{\partial \xi} + \lambda(t,\xi)w \\ &+ p\left(t, \xi, \int_0^1 h(\xi, s)w(t,s)ds, \int_0^1 k(\xi,s)\frac{\partial w}{\partial t}(t,s)ds\right) + u(t,\xi),\end{aligned}$$

with $u(t,\xi) \in U(t,\xi)$.

References

[1] Cardinali T., Duricchi G., Taddei V. *On second order nonautonomous semilinear differential inclusions with the nonlinear part depending on speed*, Submitted.

[2] Cardinali T., Duricchi G., Taddei V. *Modelling impulsive wave propagation over a finite/infinite time interval*, Submitted.

[3] Kozak M., *A fundamental solution of a second-order differential equation in Banach space*, Univ. Iagel. Acta Math., **32**, 275-289, 1995.