

On the Existence of Positive Solutions to Boundary Value Problems with Riemann-Liouville Fractional Derivatives

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Abstract

We investigate the existence, non-existence, uniqueness, and multiplicity of positive solutions to the following problem

$$\begin{cases} D_{0+}^{\alpha} u + h(t)f(u) = 0, & 0 < t < 1, \\ u(0) = u(1) = 0, \end{cases}$$

where D_{0+}^{α} is the Riemann-Liouville fractional derivative of order $\alpha \in (1, 2]$. Firstly, by characterizing the first eigenvalue $\lambda_1(\alpha)$ of the associated eigenvalue problem, we establish the existence of positive solutions for both sublinear and superlinear cases relative to $\lambda_1(\alpha)$, thereby extending previously known results. Secondly, we address the uniqueness of these solutions. In the sublinear case, we impose certain monotonicity conditions on f , while for the superlinear case, we assume a specific condition on h to ensure uniqueness at $\alpha = 2$. For values of α near 2, uniqueness is established by leveraging the non-degeneracy of the unique solution. Finally, we apply this methodology to Hénnon-type problems to demonstrate the existence of at least three positive solutions. This is a joint work with Inbo Sim (University of Ulsan).

References

- [1] I. Sim and S. Tanaka, Positive solutions for fractional-order boundary value problems with or without dependence of integer-order ones, *Fract. Calc. Appl. Anal.* **29** (2026), no. 1, 66–100.