

# Maximum Principle for the Hill Equation: Eigenvalue Approach

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## Abstract

We consider the linear second-order periodic problem

$$-u'' + p(t)u = h(t), \quad u(0) = u(\omega), \quad u'(0) = u'(\omega)$$

with a sign-changing coefficient  $p$  and provide new possible answers to one of its related fundamental questions:

*For which  $p$  does a nonnegative  $h$  result in a nonnegative  $u$ ?*

In other words, for which  $p$  does the maximum principle hold. We can find several necessary and/or sufficient conditions on the coefficient  $p$  in the extensive relevant literature. Optimality and especially applicability and verifiability of these conditions are crucial for further studies of related nonlinear problems.

Inspired by our previous results concerning the fourth-order problems, we state alternative series of (optimal and/or verifiable) conditions on  $p$  based on the principal weighted eigenvalue of the corresponding linear operator. We also present particular examples and comparison with some so far known conditions.

## References

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