Spectral Characterization of the Constant Sign Derivatives of Green's Functions Related to Two-Point BVPs

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Abstract

This talk is devoted to the study of the following n-order linear differential equation

$$T_n[M]\,u(t) := u^{(n)}(t) + M\,u(t) = 0\,,\ t \in [a,b]\,, \tag{1}$$

where $M \in \mathbb{R}$ is a real parameter, coupled to the two-point boundary conditions

$$u^{(\sigma_1)}(a) = \quad \cdots \quad = u^{(\sigma_k)}(a) \quad = 0, \tag{2}$$

$$u^{(\varepsilon_1)}(b) = \quad \cdots \quad = u^{(\varepsilon_{n-k})}(b) = 0, \tag{3}$$

with $k \in \{1,\dots,n-1\}$ and the sets of indices $\{\sigma_1,\dots,\sigma_k\}$, $\{\varepsilon_1,\dots,\varepsilon_{n-k}\} \subset \{0,\dots,n-1\}$, satisfying that

$$0 \le \sigma_1 < \sigma_2 < \cdots < \sigma_k \le n-1\,, \quad 0 \le \varepsilon_1 < \varepsilon_2 < \cdots < \varepsilon_{n-k} \le n-1\,.$$

More concisely, it is characterized the set of parameters M for which certain partial derivatives of the Green's function g_M, related to problem (1)–(3), are of constant sign, without relying on its explicit expression. It is proved that this set is an interval whose endpoints are characterized by the first eigenvalues of the operator $T_n[M]$ coupled to suitable boundary conditions. As a consequence, some sufficient conditions are provided ensuring that these derivatives cannot be nonpositive (or nonnegative). The obtained results are compiled in [1] and extend, for the particular case of operator $T_n[M]$, the ones proved in [2] for the constant sign of the Green's functions of the general n-th order linear differential operator, with non constant coefficients, and coupled to the boundary conditions (2)–(3)

References

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[2] Cabada, A., Saavedra, L., Characterization of constant sign Green's function for a two-point boundary-value problem by means of spectral theory. Electron. J. Differential Equations, 2017 (2017), 146, 1–95.