

Nonlinear Effects in Linear Delay Equations and Linear Approach to Nonlinear Models with Delay Mortality

Elena Braverman

Department of Mathematics and Statistics, University of Calgary,
Calgary, Alberta, Canada
MAELENA@UCALGARY.CA

Abstract

In the first part of the talk, we consider a scalar linear mixed differential equation with several terms, both delayed and advanced arguments and a bounded right-hand side. Assuming that the deviations of the argument are bounded, we present sufficient conditions when there exists a unique bounded solution on the positive half-line. Explicit tests are obtained when a bounded solution of a homogeneous equation decays exponentially. Existence of exponentially decaying solutions for this class of differential equations has not been studied before. We show that the standard approach when convergence of all solutions is stated does not work for mixed equations; in addition to an exponentially decaying, there may be a growing solution. All the coefficients and the mixed arguments are assumed to be Lebesgue measurable functions, not necessarily continuous. Though the equation is linear, some properties, as well as the methods applied, are more typical for nonlinear models, for example, fixed-point theorems used in the proofs.

In the second part, we explore existence of positive solutions, persistence, and boundedness of solutions for the Nicholson blowflies model with delayed mortality term $-\delta N(h(t))$. Two global stability tests for the positive equilibrium are obtained based on a linearized global stability method, reducing stability of a non-linear model to a specially constructed linear equation. The first test extends the absolute stability result to the case of delayed mortality, and the second one is delay-dependent. This is a joint work with Leonid Berezhansky (Ben-Gurion University of the Negev, Israel) and Sandra Pinelas (Academia Militar, Amadora, Portugal).

References

- [1] L. Berezhansky, E. Braverman, Nicholson's blowflies differential equations with a small delay in the mortality term, *Nonlinear Analysis: Real World Applications* **81** (2025), Paper # 104193, 10 pp.
- [2] L. Berezhansky, E. Braverman, and S. Pinelas, Exponentially decaying solutions for models with delayed and advanced arguments: nonlinear effects in linear differential equations, *Proc. AMS* **151** (2023), 4261–4277.