

Blow-up and Convergence of Solutions to a Differential Equation with Delayed Negative Feedback

Yukihiko Nakata

College of Science and Engineering, Aoyama Gakuin University
YNAKATA@MATH.AOYAMA.AC.JP

Abstract

There have been studies on the existence of finite-time blow-up solutions for delay differential equations, see [1, 2, 3, 4, 5]. For those equations, identifying blow-up solutions, estimating their blow-up times and characterizing the blow-up behaviour are generally challenging problems. In this talk, we discuss the dynamics of the following delay differential equation:

$$x'(t) = x(t)^2 - x(t - \tau), \quad (1)$$

where $\tau > 0$. Eq (1) is a simple model incorporating “self-amplification” and a delayed negative feedback. We discuss the blow-up phenomena of (1), comparing it to (1) with $\tau = 0$, and provide sufficient conditions on the initial function for finite-time blow-up solution. We also investigate the convergence of the solution to the zero solution. This study is based on the collaboration work with T. Ishiwata (Shibaura Institute of Technology).

References

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