Connecting Orbits for Some Mackey–Glass Type Equations

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Abstract

Consider the delay differential equation

$$y'(t) = -ay(t) + bf(y(t-1)) \tag{1}$$

where $b > a > 0$, and $f(\xi) = \frac{\xi^k}{1+\xi^n}$ with $k > 0$ and $n > 0$. The case $k = 1$ is the famous Mackey–Glass equation, the case $k > 1$ appears in population models with the Allee effect, and $k \in (0, 1)$ arises in some economic growth models.

We consider an unstable stationary point of (1), and its unstable set. The unstable set contains connecting orbits from the stationary point to a subset $\mathcal{A}$ of the phase space. The aim is to describe the structure of $\mathcal{A}$, which can be complicated for some parameter values, see [1].

In case $k \in (0, 1)$ and the stationary point is 0, the invariant manifold technique is not applicable, since the nonlinearity f is not differentiable at 0. Nevertheless, we construct a positive heteroclinic solution from 0, in this singular case as well.

References

[1] G. Benedek, T. Krisztin, Connecting orbits for delay differential equations with unimodal feedback, J. Diff. Equations, Volume 465, 2026, 114252, ISSN 0022-0396, https://doi.org/10.1016/j.jde.2026.114252.