Spectral Element Methods for Boundary-Value Problems of Functional Differential Equations

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Abstract

We prove convergence of the spectral element method for piecewise polynomial collocation applied to periodic boundary value problems BVP for functional differential equations. In particular, we prove that the numerical collocation solution approximates the true solution with accuracy of order $\mathrm{e}^{-\eta m}$ for some $\eta > 0$ and increasing degree m of the polynomials for a case that is common in applications: differential equations where the right-hand side depends on a finite number of delayed arguments with parametric delays and real analytic coefficients. For state-dependent delays the spectral element method also converges under mild regularity assumptions, but the geometric convergence of the collocation solution depends on the properties of the true solution, which may in general not be real analytic even for analytic coefficients.
We introduce the concept of extended local Lipschitz continuity of the right-hand side, which allows us to extend our convergence results to the case of state-dependent delays. In particular, we can arrive at the same geometric convergence statement by assuming, in this case, analyticity of the solution rather than of the right-hand side.