Slow-Fast Solutions of Abel Equations

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Abstract

In this talk, we present an overview of the new results we have obtained concerning trigonometric slow–fast Abel equations.

Specifically, we study the equation

$$\varepsilon \frac{dx}{d\theta} = A_\varepsilon(\theta)x^3 + B_\varepsilon(\theta)x^2,$$

or equivalently the system

$$\begin{cases} \dot{\theta} = \varepsilon, \\ \dot{x} = A_\varepsilon(\theta)x^3 + B_\varepsilon(\theta)x^2. \end{cases}$$

Our main goal is to characterize the limit cycles near the slow–fast regime, proving along the way the existence of canard periodic solutions. To achieve this, we analyze the behavior of solutions both near the critical curves and near infinity. This analysis requires the use of the blow-up technique.

As a corollary of this study, we recover the result in [1], where the authors proved that the maximum number of limit cycles of the linear trigonometric Abel equation is three.

References

[1] X. Yu, J. Huang, C. Liu. Maximum number of limit cycles for Abel equation having coefficients with linear trigonometric functions. Journal of Differential Equations, 410 (2024), 301–318.