**The Hunter–Saxton Equation - Wave Breaking Leads the Way**

Katrin Grunert

NTNU Norwegian University of Science and Technology, Trondheim, Norway
KATRIN.GRUNERT@NTNU.NO

**Abstract**

Solutions of the Hunter–Saxton equation might enjoy wave breaking in finite time. This means that even classical solutions in general do not exist globally, but only locally in time since their spatial derivative might become unbounded in finite time, while the solution itself remains bounded. In addition, energy might concentrate on sets of measure zero when wave breaking occurs. As a consequence the prolongation of solutions beyond wave breaking is non-unique and depends heavily on how the concentrated energy is manipulated.

In this talk, we will focus on $\alpha$-dissipative solutions, i.e., solutions where the energy is manipulated at breaking time by taking out an $\alpha$-part of the concentrated energy. Guided by analytical methods and results, we will present a numerical method for $\alpha$-dissipative solutions, which can handle any $\alpha \in [0, 1]$ and in particular the two most prominent type of solutions: conservative, i.e., $\alpha = 0$, and dissipative, i.e., $\alpha = 1$. Convergence is obtained for any admissible initial data, while a convergence rate is derived under some additional constraints on the initial data.

# References

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[2] T. Christiansen and K. Grunert, Rate of convergence for numerical $\alpha$-dissipative solutions of the Hunter–Saxton equation, Numer. Math. 157, 1643–1694 (2025).