Lie Group Analysis, Group-Invariant Solutions and Conservation Laws of a Family of Coupled Nonlinear Schrödinger Equations

Vassil Vassilev

Institute of Mechanics, Bulgarian Academy of Sciences
Acad. G. Bonchev St., Building 4, 1113 Sofia, Bulgaria
VASILVAS@IMBM.BAS.BG

Abstract

In this talk we present results concerning the symmetry properties of the following family of coupled nonlinear Schrödinger equations

$$i\frac{\partial u}{\partial t} + p_1\frac{\partial^2 u}{\partial x^2} + p_{11}u + p_{12}v + \left(q_{11}|u|^2 + q_{12}|v|^2\right)u + q_{13}v^2u^* + q_{14}u^2v^* = 0,$$

$$i\frac{\partial v}{\partial t} + p_2\frac{\partial^2 v}{\partial x^2} + p_{21}u + p_{22}v + \left(q_{21}|u|^2 + q_{22}|v|^2\right)v + q_{23}u^2v^* + q_{24}v^2u^* = 0,$$

under Lie groups of point transformations of the respective space of real-valued independent variables (t, x) and complex-valued dependent variables (u, v). Here, p_1, p_2, p_{11}, p_{12}, p_{21}, p_{22}, q_{11}, q_{12}, q_{21}, q_{22}, q_{13}, q_{23}, q_{14}, q_{24} are real numbers and the asterisk $(^*)$ denotes the complex conjugate of a complex function. Systems of equations of this type are used to describe the propagation of waves (polarized optical pulses in weakly birefringent waveguides, for instance) in nonlinear, dispersive and dissipative media (see, e.g., [1]). This family comprises and slightly extends the set of the systems of coupled nonlinear Schrödinger equations whose group properties have been studied in [2, 3, 4, 5].

First of all, the current study focuses on determining the point Lie symmetry groups admitted by the system of equations under consideration with respect to the set of parameters involved, i.e., the related group classification problem is solved. Then, several types of group-invariant solutions are identified and presented in explicit form. Finally, cases in which systems belonging to the regarded family of evolution partial differential equations admit an exact variational formulation are considered. Their variational and divergence symmetries are determined, and the corresponding conservation laws are obtained via Noether's theorem (c.f. [6]).

References

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