

Use of Repeated Integration in Efficient Algorithms for Numerical Analysis of Selected Problems of Computational Mechanics

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Abstract

Standard numerical methods for numerical analysis of such problems as structural vibrations or elastic wave propagation in computational mechanics are frequently inaccurate or rather complicated; cf. citeVala19,Kolm24. Thus, it is reasonable to seek alternative approaches for handling such problems. A promising computational technique, namely for all Cauchy initial value problems, comes from the Cauchy formula for repeated integration, introduced by [3, 4]. For computation of repeated integrals and derivatives, appropriate modifications of Newton-Cotes formulae can be developed, as well as those for Lobato, Radau, etc., quadrature ones; see [5]. In general, the explicit methods of Runge-Kutta type, referring to the Butcher series by [6], as an algebraic tool for analysing solutions to ordinary differential equations or their systems, including approximate solutions, can be combined with repeated integration effectively. For partial differential equations of evolution, this approach is applicable, too. It can work, e.g., with the finite element technique from [2] in the 3-dimensional Euclidean space, which converts the original problem to the analysis of a sparse system of ordinary differential equations. In addition to existence and convergence considerations, a practical comparison of various numerical approaches will be presented.

References

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