Exact Low Multiplicity Results for Positive Solutions of Some Sublinear Problem with Indefinite Weights under Robin Boundary Conditions

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Abstract

In this talk, we consider positive solutions of the sublinear elliptic equation with Robin boundary conditions

$$\begin{cases} -\Delta u = a(x)u^q & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = \alpha u & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 1$, is a smooth bounded domain, $a \in C^\theta(\overline{\Omega})$, $\theta \in (0,1)$, changes sign, $q \in (0,1)$, $\alpha \geq 0$ is a parameter, and ν is the unit outer normal to $\partial\Omega$. We focus on positive solutions lying in $P^\circ = \{u \in C(\overline{\Omega}) : u > 0 \text{ in } \overline{\Omega}\}$. First, we observe that if there exists a positive solution in P°, then $\int_\Omega a < 0$. Next, in a specific case that a and q satisfy that $\int_\Omega a \simeq 0$ and $q \simeq 1$, respectively, we prove the existence of $\alpha_s > 0$ such that:

- at $\alpha = 0$ and α_s, there exists a *unique* positive solution in P°;
- for every $\alpha \in (0, \alpha_s)$, there exist *exactly two* positive solutions ordered in P°;
- for any $\alpha > \alpha_s$, there is *no* positive solutions in P°.

The analysis uses bifurcation methods and the spectral theory for the associated linearized eigenvalue problem. This is a joint work with Uriel Kaufmann and Humberto Ramos Quoirin.

References

[1] U. Kaufmann, H. Ramos Quoirin, K. Umezu, Nonnegative solutions of an indefinite sublinear Robin problem II: local and global exactness results. Israel J. Math., **247**, (2022), 661–696.