About the Occurrence of Akhmediev Breathers in General Dispersive Systems

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Abstract

The Nonlinear Schrödinger Equation (NLS equation) can be derived by multiple scaling perturbation analysis for the description of slow modulations in time and space of the envelope of spatially and temporarily oscillating wave packets, as they appear in nonlinear optics, water wave theory, plasma physics, waves in DNA, Bose-Einstein condensates, etc.

The NLS possesses a number of special solutions such as solitons, Peregrine solutions, Kuznetsov-Ma solitons, and Akhmediev breathers. In previous publications it has been shown that there are solutions of the original physical systems which behave as predicted by the NLS equation. The existing theorems cover the solitons, cf. [1, 2] and recently also the Peregrine solutions and the Kuznetsov-Ma solitons [3], but not the Akhmediev breathers, which are solutions who are spatially periodic and localized in time above a time-periodic spatially constant background state [4]. In this talk we present new methods to bridge this gap, where the main point of consideration in the choice of a suitable space for our approximations.

References

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