Extinction, Persistence and Pattern Formation in Non-Linear Models with Non-Local Dispersal on Flat Landscapes

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Abstract

In this talk, we first explore a general mathematical framework for non-linear models describing the evolution of species density through non-local dispersal. We focus on the spectral analysis of non-local operators that serve as scaled analogues to the classical Laplacian with Neumann or Dirichlet boundary conditions. We establish explicit formulas and asymptotic behavior for eigenvalues and eigenfunctions, which allow us to determine the well-posedness of the system, the existence of invariant regions, and the long-time asymptotic behavior of solutions, including convergence to the mean mass and the construction of spatially heterogeneous steady states [2].

Building upon these theoretical foundations, we investigate the dynamics of vegetation patterns in water-limited ecosystems using a generalized Klausmeier model. By incorporating non-local plant dispersal within finite habitats, we identify rigorous criteria for vegetation survival and extinction governed by the trade-off between local growth and boundary losses. Our results reveal the existence of a critical patch size and a critical maximal biomass density, below which the population inevitably collapses to a desert state. Furthermore, we demonstrate that non-local dispersal mechanisms—particularly those characterized by fat-tailed kernels—significantly enhance ecosystem resilience, allowing vegetation to persist in smaller and more fragmented habitats than predicted by classical local reaction-diffusion models [1].

References

[1] M. Tadej, R. Martinez-Garcia, M. Hecht, *Extinction and persistence criteria in non-local Klausmeier model of vegetation dynamics on flat landscapes*, Physica D: Nonlinear Phenomena (submitted, 2026).

[2] M. Tadej, *Long time behaviour of solutions to non-local and non-linear dispersal problems*, Journal of Differential Equations, 416 (2025), 2043–2064.