Symbolic Dynamics in Pseudospectral Projection of Delay Differential Equations

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Abstract

We study the following class of Delay Differential Equations (DDEs):

$$\begin{aligned} x'(t) &= f(x(t), x(t-\tau)) && t \geq 0 \\ x(t) &= \psi(t) && t \in [-\tau, 0], \end{aligned}$$

with $\tau > 0$ fixed, $x \in \mathbb{R}^d$, $\psi \in \mathcal{C}^0([-\tau, 0], \mathbb{R}^d) =: \mathcal{C}$. While significant work has been done on understanding the dynamics of such systems–most notably regarding chaos in the Mackey-Glass equation[1]–the infinite-dimensional nature of DDEs presents some challenges for carrying out the rigorous computations and obtaining estimates good enough for computer-assisted proofs remains a challenge, see [2] and references therein.

This talk will present a recent attempt to reduce the number of unknown factors affecting the quality of rigorous numerics by using a pseudospectral approximation [3], which reduces the DDE to a relatively small system of ODEs while preserving numerically observed dynamical features. Due to the low-dimensionality of the resulting approximation and its great theoretical accuracy, the computations are less demanding and can be done using known tools, such as CAPD [4], to efficiently verify some dynamical phenomena that closely mirror those of the full system. We present a proof of existence of symbolic dynamics in the ODE approximating DDE and we discuss lessons learned that might be used to guide a similar proof for the full DDE system. **Acknowledgments:** the author acknowledge the support of Polish National Science Center (NCN) grant no. 2023/49/B/ST6/02801.

References

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