

A Compact Higher-Order Formulation for Unsteady Two-Dimensional Convection–Diffusion Systems with RK-Type Time Integration

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Abstract

The work is concerned with developing a high-order compact (HOC) finite-difference method for the two-dimensional unsteady convection–diffusion equation in conjunction with the classical fourth-order Runge–Kutta (RK4) strategy for time advancement on a nine-point stencil. The fully explicit time integration enables straightforward implementation of the HOC-RK4 formulation, ensuring computational efficiency without the need to solve large linear systems. A detailed von Neumann analysis demonstrates the method’s robustness under highly convective conditions. Its accuracy and versatility are further evaluated using benchmark cases that include Dirichlet, Neumann, and periodic boundary conditions. In a diffusion-dominated test of a Gaussian pulse, the method effectively captures both diffusion and anti-diffusion transport without introducing artificial oscillations. Numerical results demonstrate fourth-order spatial accuracy and stability while employing RK4 time integration for Reynolds number (Re) up to 7500. At high Re s, lid-driven cavity simulations accurately resolve secondary and tertiary vortices as well as complex recirculation patterns, demonstrating their suitability for large-scale computations. We examine the flow around a diamond-shaped cylinder, where our method successfully reproduces the well-known α and β phenomena in the wake at higher Re . In the case of the impulsively started circular cylinder, a recognized unsteady laminar benchmark, the method accurately captures the temporal development of primary, secondary, and tertiary vortices, revealing intricate wake dynamics. The technique effectively identifies sub- α and sub- β phenomena, including a previously unreported simultaneous occurrence with the α -phenomenon at $Re = 3000$, showcasing its robustness in capturing complex flow behaviors. The computed flow characteristics align closely with experimental results, and in both above scenarios, the predicted vortex patterns and symmetry breaking correspond well with reference studies.