Levinson-Pliss Theorem for Periodic Lattice Dynamical Systems

David Cheban and Andrei Sultan

Moldova State University, Chisinau, Republic of Moldova
david.ceban@usm.md, andrew15sultan@gmail.com

Abstract

In this talk we study the compact global attractors of periodic dynamical systems. We establish the relation between periodic compact dissipative dynamical systems and the compact dissipative discrete dynamical (Poincare map) generated by given periodic system. Using this relation we establish some important facts for compact dissipative periodic systems. We apply the obtained general results for the periodic lattice dynamical systems [1] of the form

$$u_i' = \nu(u_{i-1} - 2u_i + u_{i+1}) - \lambda u_i + F(u_i) + f_i(t) \ (i \in \mathbb{Z}). \tag{1}$$

In particular we show that under some conditions the periodic lattice dynamical system (1) admits a compact global attractor which contains at least one periodic solution of the system (1).

We have established the following result.

Theorem. Assume that the following conditions hold:

1. the functions $f_i \in C(\mathbb{R}, \mathbb{R})$ (for all $i \in \mathbb{Z}$) are τ-periodic;
2. the function $F \in C(\mathbb{R}, \mathbb{R})$ is Lipschitz continuous on bounded sets and $F(0) = 0$;
3. there exists a positive number α such that $sF(s) \leq -\alpha s^2$ for any $s \in \mathbb{R}$.

Then the lattice dynamical system (1) is compact dissipative [2], i.e., there exists a nonempty compact subset $I \subset \ell_2$ possessing the following properties:

1. $P(I) = I$, i.e., the set I is invariant w.r.t. the Poincare transformation P;
2. I is orbitally stable;
3. the set I is globally attracting;
4. the Poincare mapping P is asymptotically compact;
5. the map P has at least one fixed point $x_0 \in I$, i.e., $P(x_0) = x_0$.

References

[1] Petr W. Bates, Kening Lu and Bixiang Wang, Attractors for Lattice Dynamical Systems. International Journal of Bifurcation and Chaos, Vol. 11, No. 1 (2001), pp.143-153.

[2] Cheban D. N. *Global Attractors of Nonautonomous Dynamical and Control Systems. 2nd Edition.* Interdisciplinary Mathematical Sciences, vol.18, River Edge, NJ: World Scientific, 2015, xxv+589 pp.