Sharp Delay-Independent Global Stability for a Lotka-Volterra System with Multiple Delays

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Abstract

Delays are known to destabilize dynamical systems and to generate complex behaviors such as oscillations and chaos, especially in nonlinear delay differential equations. Even for the classical Lotka-Volterra predator-prey model, whose global dynamics are well understood in the absence of delay, the introduction of time lags can drastically complicate the qualitative behavior, and sharp characterizations of global stability remain a challenging problem in functional differential equations.

This talk considers a symmetric Lotka-Volterra predator-prey system with four discrete delays appearing in the intraspecific interaction terms, where both delayed and undelayed feedbacks are incorporated and a unique positive equilibrium exists under natural assumptions. Here, the parameter a represents the strength of instantaneous self-limitation, while α and β denote the strengths of delayed interaction effects. The main result establishes a necessary and sufficient condition for delay-independent global asymptotic stability: the positive equilibrium is globally asymptotically stable for all nonnegative delays if and only if $|\alpha|+|\beta| \leq a$ ([1]). The sufficiency is proved by constructing an appropriate Lyapunov functional and applying LaSalle's invariance principle, while the necessity follows from a local analysis near the equilibrium.

A comparison with the corresponding two-delay system, for which the sharp stability condition is given by $\sqrt{\alpha^2+\beta^2} \leq a$ (see [2]), reveals a clear discrepancy between the two settings. This highlights that the global dynamics of delayed Lotka-Volterra systems depend sensitively not only on the magnitude of interaction coefficients but also on how delays are distributed within the nonlinear structure.

References

[1] Y. Saito, Sharp global stability in a delayed Lotka-Volterra predator-prey system: a delay-placement discrepancy, submitted.

[2] Y. Saito, T. Hara, and W. Ma, Necessary and sufficient condition for permanence and global stability of Lotka-Volterra system with two delays *J. Math. Anal. Appl.*. **236** (1999) 534-556.