

Localization of Solutions in Annular Regions Times Conical Shells for Systems with Sign-Changing Nonlinearities

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Abstract

In this talk, we provide an existence result for abstract nonlinear operator systems of the form

$$\begin{cases} u_1 = T_1(u_1, u_2), \\ u_2 = T_2(u_1, u_2), \end{cases}$$

where $T = (T_1, T_2) : \mathcal{A} \times \mathcal{K} \subset X \times Y \rightarrow X \times Y$ is a compact map and $\mathcal{A}$ and $\mathcal{K}$ are an annular and a conical region in the normed spaces X and Y , respectively. To localize the solutions of the system, compression-expansion type conditions are imposed in each component of the operator T . Our approach [1] relies on the topological methods and, more concretely, it is based on the multiplicativity property of the Leray-Schauder fixed point index.

As an application, a system of ODEs with two point boundary conditions is studied. We consider the case in which one of the nonlinearities in the system is nonnegative whereas the other is sign-changing. In this setting, the solution is located within the product of annular regions and conical shells and thus both of its components are nontrivial.

References

- [1] G. Infante, G. Mascali and J. Rodríguez-López, An existence result in annular regions times conical shells and its applications to nonlinear Poisson systems, *Ann. Mat. Pura Appl.*, to appear (2026).