Fractional SIR Model with Distributed Contacts

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Abstract

We propose a spatial epidemiological model extending the classical SIR framework by incorporating spatial dependence through a distributed contacts (DC) approach [1]. Spatially non-local interactions arising from long-range contacts are incorporated via a superdiffusive term, leading to a fractional DC–SIR model governed by the fractional Laplacian in the Riesz integral sense [2].

Motivated by empirical evidence that human mobility is more accurately characterised by superdiffusive processes than by classical diffusion, owing to long-distance movements (Lévy flights) [3], we compare the numerical results obtained from the fractional and classical diffusive DC–SIR models and analyse spatial dependence through variations in the fractional order α, interpreted as a contact distribution parameter.

Numerical simulations are performed using a method-of-lines approach, combining finite-difference spatial discretisation with a third-order Runge–Kutta time integration scheme. The basic reproduction number R_0 is defined and numerically approximated, acting as a threshold for disease extinction or persistence. The convergence order of the numerical methods used to approximate the solution of the DC-SIR model and R_0 is illustrated through numerical examples.

References

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