Data-Driven Stabilization Parameters for CDR Equations on General Meshes

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Abstract

Numerical simulation of fluid flows frequently requires solving convection–diffusion–reaction (CDR) equations, which model the transport of quantities such as temperature, concentration, or vorticity in a moving fluid. In convection-dominated regimes—where advective transport overwhelms diffusion—standard finite element methods often suffer from spurious oscillations and unstable solutions. The Streamline Upwind/Petrov–Galerkin (SUPG) method addresses this by augmenting the standard Galerkin variational formulation with a residual-based stabilization term controlled by an element-wise parameter δ_K. Consider the CDR problem

$$-\varepsilon\,\Delta u + \mathbf{b}\cdot\nabla u + cu = f \quad \text{in } \Omega, \qquad u = u_b \quad \text{on } \partial\Omega, \tag{1}$$

where u is the transported variable, $\mathbf{b}$ the velocity field, ε the diffusion coefficient, c a reaction rate, and f a source term. The SUPG method seeks $u_h \in V_h$ such that

$$a(u_h, v_h) + \sum_{K\in\mathcal{T}_h} \delta_K \big(-\varepsilon\,\Delta u_h + \mathbf{b}\cdot\nabla u_h + c\,u_h - f,\ \mathbf{b}\cdot\nabla v_h\big)_K = 0 \quad \forall\, v_h \in V_h, \tag{2}$$

where $a(\cdot,\cdot)$ is the standard Galerkin bilinear form, v_h the test function, and $\delta_K > 0$ the stabilization parameter that controls the amount of artificial diffusion added along the streamlines. Selecting an appropriate value of δ_K is critical: a too small value leads to oscillatory solutions, while a too large value introduces excessive numerical diffusion. Classical choices perform poorly on most of the mesh geometries.

In this work, we extend our machine learning–based framework for computing optimal SUPG stabilization parameters to unstructured and anisotropic meshes, which are far more prevalent in practical CFD applications than the structured setting. A neural network model is trained within the SUPG framework to learn local flow and mesh features and to output element-wise optimal parameters δ_K. The network is trained on a diverse range of mesh configurations, enabling it to generalize robustly to structured, unstructured, and strongly anisotropic meshes alike. Comparative numerical experiments demonstrate that the proposed ML-enhanced SUPG method yields more accurate solutions than the standard SUPG method on all tested mesh types, successfully suppressing spurious oscillations to a degree consistent with the quality obtained on structured grids. These results underscore the potential of data-driven stabilization strategies for delivering reliable, oscillation-free finite element solutions to convection-dominated problems on general computational meshes.