Delay-Induced Instabilities in Neural Field Equations

Mónika Polner

Bolyai Institute, University of Szeged, Hungary
POLNER@MATH.U-SZEGED.HU

Abstract

We investigate neural field equations with transmission delays as infinite-dimensional dynamical systems, analyzing their spectral properties, bifurcation scenarios, and symmetry-induced mechanisms of pattern formation. On bounded two-dimensional domains with exponential connectivity kernels, the characteristic equation is reformulated as a linear partial differential equation, permitting a complete characterization of the point spectrum and resolvent for single-exponential kernels. This approach yields explicit conditions for Hopf bifurcations and an analytical expression for the first Lyapunov coefficient, providing a rigorous description of delay-induced oscillatory instabilities [1].

Building on this foundation, we further study two-population systems on the sphere incorporating both transmission delays and diffusive coupling, following the framework of [2]. By employing spherical harmonic decompositions in combination with equivariant bifurcation theory, we derive analytical conditions for the onset of bifurcations and determine the associated normal form coefficients up to third order. The interplay between delay and diffusion mechanisms is shown to drive symmetry-breaking Hopf bifurcations and the selective stabilization of particular spherical harmonic modes.

A space–time discontinuous Galerkin method supports the theoretical analysis, [3].

References

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