

Some Oscillation Criteria for the Second-Order Linear Advanced Differential Equations

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Abstract

On the half-line $\mathbb{R}_+ = [0, +\infty[$, we consider the second-order linear differential equation with argument deviation

$$u''(t) + p(t)u(\sigma(t)) = 0,$$

where $p: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a locally Lebesgue integrable function and $\sigma: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a continuous function such that $\sigma(t) \geq t$, for $t \geq 0$.

New oscillation criteria for solutions to the considered equation are presented. These criteria extend and build upon those established in [3]. The proofs are based on the Riccati technique and suitable estimates of nonoscillatory solutions. Some of the results may be regarded as an analogy of the Hille-Nehari type oscillation criteria for ODEs introduced in [1] and [2].

References

- [1] Hille, E. *Non-oscillation theorems*, Trans. Amer. Math. Soc. **64** (1948), No. 2, 234–252.
- [2] Nehari, Z. *Oscillation criteria for second-order linear differential equations*, Trans. Amer. Math. Soc. **85** (1957), No. 2, 428–445.
- [3] Opluštil, Z. *Oscillation criteria for the second-order linear advanced differential equation*, Applied Mathematics Letters **157** (2024), No. 157.