

Optimal Time-Adaptivity for Parabolic Problems with applications to Model Order Reduction

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Abstract

Since the first optimality proofs for adaptive mesh refinement algorithms in the early 2000s, the theory of optimal mesh refinement for PDEs was inherently limited to stationary problems. The reason for this is that time-dependent problems usually do not exhibit the necessary coercive structure that is used in optimality proofs to show a certain quasi-orthogonality, which is crucial for the theory. Recently, by using a new equivalence between quasi-orthogonality and inf-sup stability of the underlying problem, it was shown that an adaptive Crank-Nicolson scheme for the heat equation is optimal under a severe step size restriction. In this work, we use this new approach towards quasi-orthogonality together with a Radau IIA method that combines the advantages of the Crank-Nicolson and implicit Euler schemes. We obtain the first adaptive time stepping method for non-stationary PDEs that is provably rate optimal with respect to number of time steps vs. approximation error. Together with a reduced basis method that leverages the Laplace transform for building tailored subspaces of reduced dimension, we obtain a very efficient method.