**Transport of Scalar Quantities Along Evolving Planar Curves and its Numerical Solution**

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**Abstract**

We investigate the dynamics of scalar quantities distributed along planar curves evolved by the constrained curvature flow. Such coupled geometric and transport processes arise naturally in a wide range of physical, chemical and biological systems, including interfacial dynamics, phase transitions, and mechanobiological modeling (see [1, 2]). In this work, we formulate a geometric framework that captures the interplay between curve evolution and scalar transport at moving interfaces. The governing system of partial differential equations is derived and analyzed, highlighting structural properties such as conservation mechanisms and curvature-driven effects (see [3, 4]). For a numerical solution, we use the flowing finite volume method in space and a higher-order time solver. We further present numerical simulations that demonstrate how the constrained curvature flow influences the redistribution and long-term behavior of scalar quantities along evolving curves. The results provide insight into the mutual interaction between geometry and transport phenomena and offer a foundation for accurate and stable computational modeling of such coupled systems.

# References

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