Multiscale Hyperbolic-Parabolic Models for Nonlinear Reactive Transport in Heterogeneously Fractured Porous Media

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Abstract

This talk concerns the asymptotic analysis of nonlinear reactive transport in a layered porous medium separated by a thin, geometrically heterogeneous fracture. The fracture is represented as an ε-thin channel whose aperture and internal obstacle pattern vary periodically on the microscale. Transport in the bulk domains is described by parabolic reaction–diffusion equations, whereas the fracture supports a convection–diffusion–reaction process with nonlinear reactions on the walls and obstacle boundaries. A key feature is that transport inside the fracture is convection dominated, with a Péclet number of order ε^{-1}.

Using multiscale asymptotics as $\varepsilon \to 0$, we derive a reduced homogenized model in which the thin fracture collapses to a flat interface. The homogenized limit consists of classical diffusion–reaction equations in the bulk domains coupled through homogenized nonlinear interface conditions and a first-order semilinear hyperbolic system posed on the flat interface. The latter encodes the effective fracture dynamics produced by the strong advection and the microscale geometry. The following aspects are addressed in this study: firstly, the well-posedness of the homogenized system is proven, including the derivation of regularity results; secondly, an explicit multiscale approximation incorporating boundary-layer correctors is constructed; and thirdly, quantitative error estimates in appropriate energy norms are established. These three aspects rigorously justify the reduced model and quantify the approximation accuracy with respect to the small parameter ε. The presentation is based on new results currently available in [1].

References

[1] Taras Mel'nyk, Sorin Pop, Christian Rohde (2026): *Multiscale hyperbolic-parabolic models for nonlinear reactive transport in heterogeneously fractured porous media*, Preprint: arXiv:2602.16439v1, https://doi.org/10.48550/arXiv.2602.16439